IUPUC OFFICE OF STUDENT RESEARCH

Bending stress analysis in a wooden beam with the construction of Mohr's circle

Introduction

Bending stress is familiar to anyone who has acted as an amateur builder, from lashing together tree branches with friends in the back yard while constructing a fort or building a truss structure out of toothpicks for a middle school competition. While relatively simple, these activities effectively demonstrated the relationships between stress, strain, material properties, and material geometry. However, with the increase of age and education comes the responsibility to build upon this foundational knowledge using practical applications. Wooden beams such as the one in this project are commonly used in the construction of homes and other simple structures. Similarly, larger and more complex structures use metal beams which operate under the same mathematical principles explored in this project. Therefore, a detailed exploration of stress can only help to sharpen a nascent engineer's understanding of material mechanics.

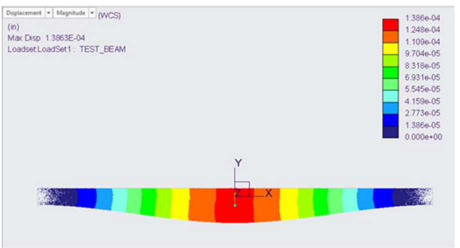
The objective of this project was to analyze and demonstrate stress in a thin wooden beam and to relate those stresses to principal stresses using Mohr's Circle.

This project consists of three stages. In the first stage, Creo computer software was used to model and analyze stresses in a wooden beam of uniform cross-section supported on either end and subjected to a downward load in the middle. In the second stage, a poplar beam was instrumented with a rosette of strain gages while a load was applied to the middle of the beam. In the third stage, data collected in the experiment was used to compute stress and construct a Mohr's circle representation of stresses in all directions.

Computer Modeling

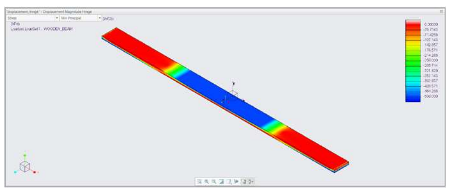
The mesh divided the beam into numerous smaller elements to facilitate the analysis. The finer the mesh, the more detailed the analysis, however it required greater computing power and longer periods of time to process.

Analysis Software: Creo Parametric Ver. 4
Beam Dimensions: 24 in x 1.5 in x 0.25 in
Beam Material: Poplar wood
Constraints: Each end
Applied Load: Concentrated, central, 5.94 N
Mesh size: 0.168 in
Mesh elements: 30,000

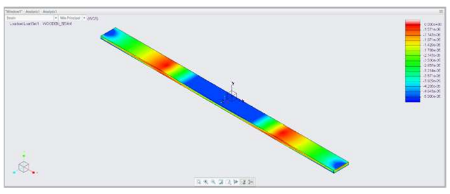


Color coded beam displacement

Beam displacement video link:
https://www.youtube.com/embed/J1SEMHmfXUc



3D image of max principal stresses

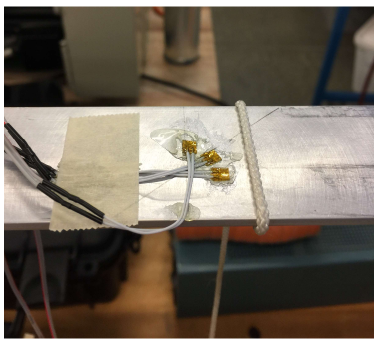


3D image of max principal strain

Experimentation

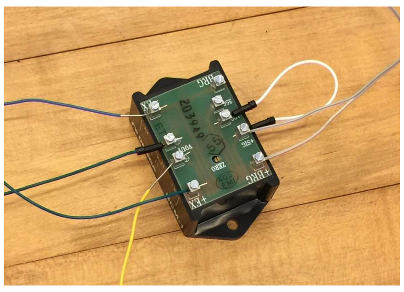
Experiment Overview:
Beam Dimensions: 24 in x 1.5 in x 0.25 in
Beam Material: Poplar wood
Load: 3.94 N
Number of Strain Gauges: 3
Strain Gauge Orientation: 0, 45, and 90 degrees
Strain Gauge Resistance: 120 Ohms
Gauge Factor: 1.51 Excitation Voltage: 5 V

Step 1: Prepare beam surface and affix strain gauges at measured angle orientations. A secondary metal beam was also prepared.

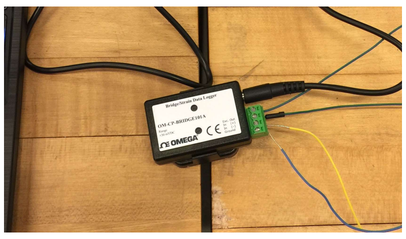


Strain gauge arrangement (metal beam)

Step 2: Wire strain gauges to bridge completion module and data logger.

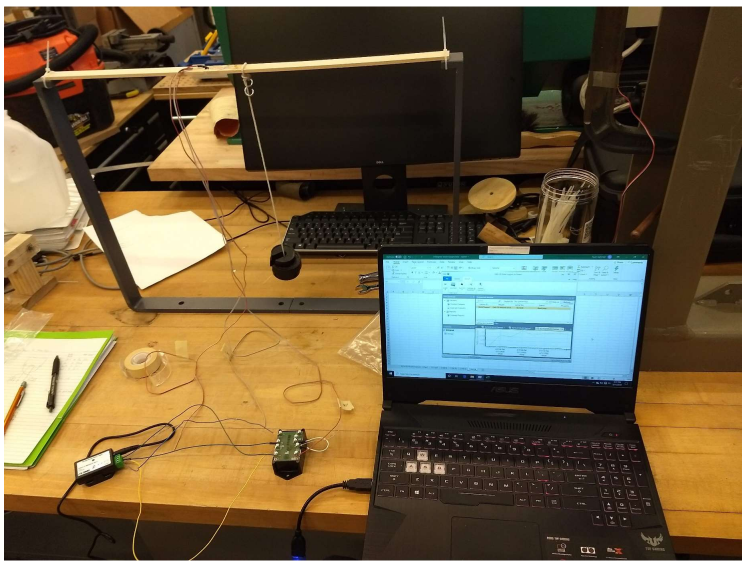


Bridge completion module



Data logger

Step 3: Apply load and measure strain.



Experimental setup

This data was used to calculate the stress in the x and y direction, as well as the shear stress.

Data Analysis

The data gathered from the strain gauges was exported into Excel for analysis.

Strain Gauge 0˚	Strain Gauge 45˚	Strain Gauge 90˚
-0.096 mV	-0.058 mV	-0.014 mV

To convert the strain gauge readouts from mV into strain, the following equation was used:

$$Strain = \frac{4*V}{GF*EV}$$

Where V is the output voltage, GF is the gauge factor of the strain gauges, and EV is the excitation voltage (5 V in this case). Additionally, the strain was converted into stress by multiplying the strain by the modulus of elasticity for poplar wood (10.9 GPa).

Finally, a system of three equations could be solved using the measured stresses and angles of the strain gauges inserted into three instances of the following equation.

$$\sigma_\theta = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}(\sigma_x - \sigma_y)\cos 2\theta + \tau_{xy}\sin 2\theta$$

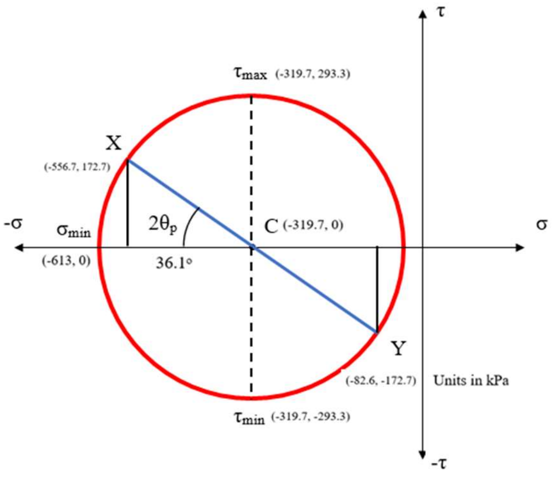
Solving the system yielded stress x = -556 kPa, stress y = -82.6 kPa, and shear stress = -172.7 kPa.

Conclusions

The stress experienced by the wooden beam while under a central load was a combination of normal and shear stress. This can be represented using Mohr's Circle, shown at right, which facilitates the calculation of stress in all directions from a single point.

Potential reasons for experimental data to differ from the computer model:
Experimental beam differed in dimensions from model due to equipment limitations.
Published modulus of elasticity of poplar wood is 10.9 GPa was used for model, however, the value was likely different in the wooden beam used in the experiment.

This project provided invaluable experience both in creating computer models and in conducting strain gauge experiments.



Mohr's Circle for poplar beam

References

[1] Beer, F., DeWolf, J., Johnston, E., and Mazurek, D., *Mechanics of Materials*, 8th ed., New York, NY: McGraw-Hill, 2019.

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